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Action at a distance as a full-value solution of Maxwell equations: The basis and application of the separated-potentials method

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The inadequacy of Liénard-Wiechert potentials is demonstrated as one of the examples related to the inconsistency of the conventional classical electrodynamics. The insufficiency of the Faraday-Maxwell concept to describe the whole electromagnetic phenomenon and the incompleteness of a set of solutions of Maxwell equations are discussed and mathematically proved. Reasons for the introduction of the so-called ''electrodynamics dualism concept'' (simultaneous coexistence of instantaneous Newton long-range and Faraday-Maxwell short-range interactions) have been displayed. It is strictly shown that the new concept presents itself as the direct consequence of the complete set of Maxwell equations and makes it possible to consider classical electrodynamics as a self-consistent and complete theory, devoid of inward contradictions. In the framework of the new approach, all main concepts of classical electrodynamics are reconsidered. In particular, a limited class of motion is revealed when accelerated charges do not radiate electromagnetic field.

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I. INTRODUCTION

In the last century, the understanding of the nature of electromagnetic phenomena was proceeding with a constant rivalry between two concepts of interaction: namely, *Newton instantaneous long-range interaction* (NILI) and *Faraday-Maxwell short-range interaction* (FMSI). Originally, owing to the fundamental works of Gauss and Ampére, all electromagnetic phenomena were related to NILI. In other words, it was understood that the interaction forces between both unmoving and moving charges at some specific time were determined by their distribution and the character of their motion at the same instant (implicit time dependence). As a matter of fact, the concept of field was merely subsidiary (it was considered in a limited sense only as an external-force field) and could be omitted entirely. On the contrary, the concept of field is primary for FMSI, but charges and currents come to be auxiliary. More fundamentally, a field is a system in its own right (has physical reality), carries energy, and fills the whole space. In accordance with Faraday-Maxwell's idea, the interaction between charged particles can be described only by the intermediary of a field as an energy-carrying physical system. Any electromagnetic perturbation must be spread through space continuously from point to point during a certain amount of time (finite spread velocity). Finally, the discovery of Faraday's law of induction (explicit time dependence of electromagnetic phenomena) and the experimental observation of electromagnetic waves seemed to confirm the field concept. Nevertheless, the idea of NILI still has many supporters. Among the physicists who have developed some theories based, in any case, on this concept, we can find names such as Tetrode and Fokker, Frenkel and Dirac, Wheeler and Feynman, and Hoyle and Narlikar [1]. This interest in the concept of NILI is explained by the fact that classical theory of electromagnetism is an unsatisfactory theory all by itself, and so there have been many attempts to modify either the Maxwell equations or the principal ideas of electromagnetism. In connection with this, we only mention some works that have tried to unify the advantage of the NILI concept with the conventional theory of field. They are the so-called ''retarded action at a distance'' theories [2–6]. The fact that all new general solutions are represented by half the retarded plus half the advanced Liénard-Wiechert solutions [7,8] of the Maxwell's equations makes it consistent with the conventional FMSI concept. On the other hand, these theories suggest the primacy of charge and use the notion of field as an external-force field such as the action at a distance theories. A single charged particle, in this approach, does not produce a field of its own, and hence has no self-energy. Thus the classical theory can be saved from some difficulties such as self-reaction force (self-interaction), the idea of a whole electromagnetic mass, etc. It turns out, however, that no one effort to straighten out the classical difficulties has ever succeeded in making a self-consistent electromagnetic theory. Moreover, the principal difficulties in Maxwell's theory do not disappear still after the quantum mechanics modifications are made. In spite of the great variety of methods applied to arrange the situation, no one theory dealing with electromagnetism had ever admitted the possibility of the simultaneous and independent coexistence of two types of interactions: NILI and FMSI. A new approach, based on this idea, has no need to modify either Maxwell equations or the basic ideas of the classical electromagnetic theory. In this work we take a complete set of Maxwell equations as correct and show that dualism of electromagnetic phenomena is an intrinsic feature. Physical and mathematical grounds for that will be given in the next sections.

II. INADEQUACY OF LIÉNARD-WIECHERT POTENTIALS: A PARADOX

The presence of a paradox in a theory does not always mean its inconsistency, but often indicates the cause of dif-

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ficulties. In this section we show one of the confusions of classical electrodynamics in describing an electromagnetic field of an accelerated charge. The attractiveness of this example consists in the way it demonstrates the difficulties of the main conventional theory and the way it leads to the idea of dualism . Let us consider a charge q moving in a laboratory reference system with a constant acceleration a along the positive direction of the X axis. An electric field created by an arbitrarily moving charge is given by the following expression obtained directly from Liénard-Wiechert potentials [9]:

$$\mathbf{E}(x,y,z,t)=q\frac{(\mathbf{R}-R\mathbf{V}/c)(1-V^2/c^2)}{(R-\mathbf{R}\mathbf{V}/c)^3}+q\frac{[\mathbf{R},[(\mathbf{R}-R\mathbf{V}/c),\dot{\mathbf{V}}/c^2]]}{(R-\mathbf{R}\mathbf{V}/c)^3}. \tag{1}$$

We note here that all values on the right-hand side (rhs) of (1) are taken in the moment of time $t_0=t-\tau$, where τ is the ''retarded time.'' We shall see that formula (1) does not satisfy the D'Alembert equation along the X axis at any time. To begin with, we note that in a free space along the X axis (except the site of a charge) an electric field component E_x satisfies the homogeneous wave equation:

$$\Delta E_x-\frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2}=0. \tag{2}$$

To find the value E_x at the moment of time t, one must take all the values on the rhs of (1) at the previous instant t_0 derived from the condition

$$t_0=t-\tau=t-\frac{R(t_0)}{c}; \quad \{(R^2=(x-x_0)^2+(y-y_0)^2+(z-z_0)^2)\} \tag{3}$$

(here (x_0,y_0,z_0) is the site of the charge at instant t_0) or from the implicit function:

$$F(x,y,z,t,t_0)=t-t_0-\frac{R}{c}=0. \tag{4}$$

Then, we have the following expression for $E_x(x,y,z,t)$:

$$E_x(x,y,z,t)=q\frac{(x-x_0-Rat_0/c)(1-a^2t_0^2/c^2)}{(R-(x-x_0)at_0/c)^3}-q\frac{a((y-y_0)^2+(z-z_0)^2)}{c^2(R-(x-x_0)at_0/c)^3}. \tag{5}$$

Substituting E_x given by (5) in the wave equation (2), one ought to calculate in any case $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$ using differentiation rules for the implicit function

$$\frac{\partial t_0}{\partial t}=-\frac{\partial F/\partial t}{\partial F/\partial t_0}; \quad \frac{\partial t_0}{\partial x_i}=-\frac{\partial F/\partial x_i}{\partial F/\partial t_0}. \tag{6}$$

As a result of the substitution of (5) into (2) one obtains (y,y_0,z,z_0 approaches zero after the differentiation):

$$\Delta E_x-\frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2}=q\frac{[3(at_0-c)(c+at_0)^2-2ac(x-x_0)]}{(at_0-c)^3(x-x_0)^4}. \tag{7}$$

In accordance with (2), the right part of (7) must be zero. This result is reasonable if we remember that wave equation (2) describes only transverse modes. In this particular case, the x component of electric field turns out to be the longitudinal one and, obviously, is inconsistent with the wave equation (2). In any other direction, solution (1) is compatible with (2). Thus, the Liénard-Wiechert potentials, as a solution of the complete set of Maxwell equations, are inadequate for describing the properties of electromagnetic field along the direction of an arbitrarily moving charge. We note here that inadequacy of Liénard-Wiechert potentials for describing the properties of relativistic fields was also shown by Whitney (see, e.g., [10]). The same singular behavior along the X-axis direction displays another important quantity. The Poynting vector represents the electromagnetic field energy flow per unit area per unit time across a given surface:

$$\mathbf{S}=\frac{c}{4\pi}[\mathbf{E},\mathbf{H}]; \quad \mathbf{P}=\frac{1}{c^2}\mathbf{S}, \tag{8}$$

where $\mathbf{S}$ is the Poynting vector, $\mathbf{P}$ is the momentum density, and $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field strength, respectively. One can easily see that expressions (8) are identically zero along the whole X axis. On the other hand, from the energy conservation law,

$$w=\frac{E^2+H^2}{8\pi}, \quad \frac{\partial w}{\partial t}=-\nabla\cdot\mathbf{S}, \tag{9}$$

we conclude that w and $\partial w/\partial t$ must differ from zero everywhere along X and there is a linear connection between w and E^2. The conflict takes place if, for instance, as the charge is vibrating in some mechanical way along the X axis, then the value of w (which is a point function like E) on the same axis will be also oscillating. Then the question arises: how does the point of observation, lying at some fixed distance from the charge on continuation of the X axis, ''know'' about the charge vibration? The presence of ''retarded time'' τ in (1) indicates that along the X axis the longitudinal perturbation should be spread with the energy transfer [contrary to (8)]. Since the vector $\mathbf{S}$ is a product of the energy density and its spreading velocity $\mathbf{v}$,

$$\mathbf{S}=w\mathbf{v}, \tag{10}$$

then either the spreading velocity $\mathbf{v}$ or the energy density w must be zero along the X axis. The first assumption puts aside the possibility of any interaction transfer. It is necessary to examine carefully the second one $(w=0)$. Maxwell's equations state that time-varying fields are transverse. In electrostatics and magnetostatics (as correct stationary approximations of Maxwell's theory), the static fields are longitudinal in the sense that the fields are derived from scalar potentials [11]. Consequently, we can assume the spreading of only longitudinal modes along the singular X-axis direction of our example capable of changing the field value at any point along this axis. In this case, according to (10), the energy of the longitudinal modes cannot be stored locally in space $(w=0)$ but the spread velocity may be any value. On the other hand, the FMSI concept forbids the spreading (not the presence) of any longitudinal electromagnetic field component in vacuum. Hence, this paradox cannot be resolved in the framework of Faraday-Maxwell electrodynamics. This simple example underlines the insufficiency of only transverse solutions of Maxwell's equations to describe full properties of electromagnetic field and leads directly to the dualism idea of *simultaneous and constant* coexistence of longitudinal (*action at a distance*) and transverse electromagnetic interactions. In the next sections one can find mathematical and physical reasons for the dualism concept that permits one to build up self-consistent classical electrodynamics. As a final remark, we make a reference to Dirac, who writes [12]: "As long as we are dealing only with transverse waves, we cannot bring in the Coulomb interactions between particles. To bring them in, we have to introduce longitudinal electromagnetic waves . . . . The longitudinal waves can be eliminated by means of mathematical transformation. Now, when we do make this transformation which results in eliminating the longitudinal electromagnetic waves, we get a new term appearing in the Hamiltonian. This new term is just the Coulomb energy of interaction between all the charged particles:

$$\sum_{(1,2)} \frac{e_1 e_2}{r_{12}}$$

. . . This term appears automatically when we make the transformation of the elimination of the longitudinal waves."

III. REASONS AND FOUNDATIONS OF THE METHOD OF SEPARATED POTENTIALS

Let us recall that a complete set of Maxwell equations is

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 4\pi\varrho, \tag{11}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{12}$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \tag{13}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}. \tag{14}$$

If this system of equations is really complete, it must describe all electromagnetic phenomena without exceptions.

It is often convenient to introduce potentials, satisfying the Lorentz the condition

$$\boldsymbol{\nabla} \cdot \mathbf{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0. \tag{15}$$

As a result, the set of coupled first-order partial differential equations (11)–(14) can be reduced to the equivalent pair of uncoupled inhomogeneous D'Alembert equations:

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -4\pi\varrho(\mathbf{r},t), \tag{16}$$

$$\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\mathbf{j}(\mathbf{r},t). \tag{17}$$

Differential equations have, generally speaking, an infinite number of solutions. A uniquely determined solution is selected by laying down sufficient additional conditions. Different forms of additional conditions are possible for the second-order partial differential equations: initial value and boundary conditions. Usually, a general solution of D'Alembert's equation is considered as an explicit time-dependent function $g(\mathbf{r},t)$. In the stationary state the D'Alembert equation is transformed into the Poisson equation, whose solution is an implicit time-dependent function $f(\mathbf{R}(t))$. Nevertheless, the conventional theory does not explain in detail how the function $g(\mathbf{r},t)$ is converted into an implicit time-dependent function $f(\mathbf{R}(t))$ (and vice versa) when the steady-state problems are studied.

Further we shall demonstrate that former solutions of Maxwell's equations are incomplete and do not ensure a continuous transition between the D'Alembert and Poisson equations' solutions, respectively. As a matter of fact, it will be shown that a mathematically complete solution of Maxwell's equations must be written as a linear combination of two nonreducible functions with implicit and explicit time dependence:

$$f(\mathbf{R}(t)) + g(\mathbf{r},t). \tag{18}$$

In the classical Faraday-Maxwell electrodynamics the Poisson equation is mathematically exact for the steady-state problems. Based on the idea of a continuous nature of electromagnetic phenomena, one could suppose that the general solution of Poisson's equation should be continuously transformed to the D'Alembert's equation solution (and vice versa) when the explicit time dependence appears (disappears). This requirement can also be formulated as a mathematical condition on the continuity of the general solutions of Maxwell's equations at every moment of time. By force of the uniqueness theorem for the second-order partial differential equations, only one solution can exist that satisfies the given initial and boundary conditions. Consequently, the continuous transition from the D'Alembert's equation solution into the Poisson's one (and vice versa) must be ensured by the continuous transition between the respective initial and boundary conditions. This is the point where the FMSI concept fails. Really, only the implicit time-dependence function $f(\mathbf{R}(t))$ can be a unique solution of Poisson's equation and boundary conditions for the external problem are to be formulated in infinity. On the other hand, the D'Alembert's equation solution is looking for only an explicit time-dependent function $g(\mathbf{r},t)$ since only that one corresponds to the classical FMSI concept as a physically reasonable solution. The boundary conditions in this case are given in a finite region. It has no sense to establish them at the infinity if it cannot be reached by any perturbation with finite spread velocity. Dealing with a large external region when the effect of the boundaries is still insignificant over a small interval of time, it is possible to consider the limiting problem with initial conditions for an infinite region (initial Cauchy's problem).

Let us consider carefully the formulation of respective boundary-value problems in a region extending to infinity [13]. There are three external boundary-value problems for Poisson's equation. They are known as the Dirichlet problem, the Neumann problem, and their combination. The mathematical problem, for instance, for the Dirichlet boundary conditions is formulated as follows. It is required to find the function $u(\mathbf{r})$ satisfying the following: (i) Laplace's equation $\Delta u=0$ everywhere outside the given system of charges (currents); (ii) $u(\mathbf{r})$ is continuous everywhere in the given region and takes the given value G on the internal surface S: $u|_S=G$; (iii) $u(\mathbf{r})$ converges uniformly to 0 at infinity: $u(\mathbf{r})\to 0$ as $|\mathbf{r}|\to\infty$.

The final condition (iii) *is essential for a unique solution*. In the case of D'Alembert's equation the mathematical problem is formulated in a different manner. Obviously, we are interested only in the problem for an infinite region (initial Cauchy's problem). So it is required to find the function $u(\mathbf{r},t)$ satisfying the following: (1) homogeneous D'Alembert's equation everywhere outside the given system of charges (currents) for every moment of time $t\geqslant 0$; (2) initial conditions in all infinite regions, as follows:

$$u(\mathbf{r},t)|_{t=0}=G_1(\mathbf{r}); \quad u_t(\mathbf{r},t)|_{t=0}=G_2(\mathbf{r}).$$

The condition (iii) about the uniform convergence at the infinity is not mentioned. We recall here that Cauchy's problem is considered when one of the boundaries is insignificant over all process time. This condition (iii) will never affect the problem and, hence, cannot be taken into account for the correct solution selecting. However, it may be formally included into the mathematical formulation of the D'Alembert's equation boundary-value problem to fulfill the formal continuity with the Poisson's equation solution at the initial moment of time. Nevertheless, this condition is already meaningless in the next instant of time since only explicit time-dependent solutions as $g(\mathbf{r},t)$ (retarded solutions with finite spreading velocity) are considered.

Thus, we underline here that the absence of the condition (iii) for every moment of time in the initial Cauchy problem does not ensure the continuous transition into the external boundary-value problem for Poisson's equation and, as a result, mutual continuity between the corresponding solutions cannot be expected by force of the uniqueness theorem. However, there is a way to solve the problem: to satisfy the continuous transition between the D'Alembert's and Poisson's equation solutions, one must look for a general solution in the form of separated functions (18) nonreducible to each other. When applied to the potentials $\mathbf{A}$ and φ this statement takes the form:

$$\mathbf{A}=\mathbf{A}_0(\mathbf{R}(t))+\mathbf{A}^*(\mathbf{r},t), \tag{19}$$

$$\varphi=\varphi_0(\mathbf{R}(t))+\varphi^*(\mathbf{r},t). \tag{20}$$

In this case, the presence of the condition (iii) in the Cauchy problem turns out to be meaningful for any instant of time, and the corresponding boundary conditions keep continuity with respect to mutual transformation.

As an additional remark, we conclude that the traditional solution of D'Alembert's equation cannot be complete, since the Faraday-Maxwell concept does not allow one to take into account the first term in (18) as valuable at any moment of time. Turning to the previous section, we see that the new solution in the form (20) is able to change the electric field component E_x along the X axis at any distance and at any time. It is quite obvious now why Liénard-Wiechert potentials (as only explicit time-dependent solution of Cauchy's problem) turned out not to be the complete solutions of Maxwell equations, and why they are not adequate to describe the whole electromagnetic field.

Let us consider again the set of Maxwell's equations (11)–(14). A pair of uncoupled differential equations can be obtained immediately for the new general solution in the form of separated potentials (19), (20) (we omit boundary conditions premeditatedly):

$$\Delta\varphi_0=-4\pi\varrho(\mathbf{r},t), \tag{21}$$

$$\Delta\mathbf{A}_0=-\frac{4\pi}{c}\mathbf{j}(\mathbf{r},t) \tag{22}$$

and

$$\Delta\varphi^*-\frac{1}{c^2}\frac{\partial^2\varphi^*}{\partial t^2}=0, \tag{23}$$

$$\Delta\mathbf{A}^*-\frac{1}{c^2}\frac{\partial^2\mathbf{A}^*}{\partial t^2}=\mathbf{0}. \tag{24}$$

The initial set of Maxwell's equations has been decomposed into two independent sets of equations. The first one, (21) and (22), answers for the instantaneous aspect (''*action at a distance*'') of electromagnetic nature while the second one, (23) and (24), is responsible for explicit time-dependent phenomena. The dualism as an intrinsic feature of Maxwell's equations is evident. The potential separation, (19) and (20), implies the same with respect to the field strengths:

$$\mathbf{E}=\mathbf{E}_0(\mathbf{R}(t))+\mathbf{E}^*(\mathbf{r},t), \tag{25}$$

$$\mathbf{B}=\mathbf{B}_0(\mathbf{R}(t))+\mathbf{B}^*(\mathbf{r},t), \tag{26}$$

where $\mathbf{E}_0$ and $\mathbf{B}_0$ are instantaneous (NILI) fields. If we see again the formula (1) based on Liénard-Wiechert potentials, then in accordance with (25) the first term must be considered without ''retarded time'' (at a given instant of time t) and the whole expression will be as follows:

$$\mathbf{E}[\mathbf{R}(t),\mathbf{R}_0(t_0),t_0]=q\frac{\mathbf{R}(1-V^2/c^2)}{R^2(1-V^2/c^2\sin^2\Theta)^{3/2}}+\mathbf{E}^*[\mathbf{a}(\mathbf{R}_0,t)], \tag{27}$$

here Θ is the angle between the vectors $\mathbf{V}$ and $\mathbf{R}$, $\mathbf{a}(\mathbf{R}_0,t_0)$ is the acceleration of the charge q in the previous moment of time $t_0=t-\tau$, and τ is the ''retarded time.'' We note that the first term in (1) is mathematically equivalent to that in (27) (see [9]). In the steady state ($\mathbf{a}=0$), the second term $\mathbf{E}^*$ must be zero, so (27) can be consistent with the requirements of the Lorentz transformation. The same approach is applicable to the Liénard-Wiechert (LW) potentials. We leave out the complete modification of LW potentials as well as an exact expression for $\mathbf{E}^*$, which, while of interest in themselves, have no direct connection with the following material.

To finish this section we conclude that NILI *must exist as a direct consequence of Maxwell equations*. According to this, both pictures, the NILI and the FMSI, have to be considered as two *supplementary descriptions of one and the same reality*. Each of the descriptions is only *partly* true. In other words, both Faraday and Newton in their external argument about the nature of interaction at a distance turned out right: instantaneous long-range interaction takes place, not *instead of*, but *along with* the short-range interaction in the classical field theory.

IV. RELATIVISTIC NONINVARIANCE OF THE CONCEPT OF ENERGY OF SELF-FIELD OF A CHARGE (SELF-ENERGY CONCEPT): MECHANICAL ANALOGY OF MAXWELL'S EQUATIONS

Maxwell's equations lend themselves to a covariant description and are in agreement with the requirements of relativity. In the previous section we have not modified the set of Maxwell's equations, we have only separated two nonreducible parts in the general solution. Hence, the usual four-vector form of the basis equations can be used. For four-vectors of separated potentials we have automatically the following expressions:

$$\Box(A_{0\mu}+A_{\mu}^{*})=-\frac{4\pi}{c}j_{\mu};\quad(\mu=0,1,2,3),\tag{28}$$

where

$$A_{0\mu}+A_{\mu}^{*}=(\varphi_0+\varphi^{*},\mathbf{A}_0+\mathbf{A}^{*});\quad j_{\mu}=(c\varrho,\mathbf{j}).\tag{29}$$

To give some substance to the above formalism we exhibit explicitly the Poisson equation for instantaneous four-vector $\mathbf{A}_{0\mu}$:

$$\Delta A_{0\mu}=-\frac{4\pi}{c}j_{\mu},\tag{30}$$

where

$$A_{0\mu}=[\varphi_0(\mathbf{R}(t)),\mathbf{A}_0(\mathbf{R}(t))].\tag{31}$$

Equation (30) is covariant also under Lorentz transformations. This is an exact consequence of (28) in the steady approximation and can be proved directly. It is supported by the well-known fact that covariance is not necessary (it is sufficient) for the relativistic invariance. Nevertheless, in the Faraday-Maxwell electrodynamics this fact was always perceived as quite odd. Actually, potentials of an unmoving charge do not have explicit time dependence. For a general Lorentz transformation from a reference system K to an inertial system K' moving with the velocity $\mathbf{v}$ relative to K, the explicit time dependence does not appear. Why do those potentials keep implicit time dependence under the Lorentz transformation? Without any approximation, the influence of a possible retarded effect is canceled itself at any time and at any distance from the moving charge. On the other hand, the conventional theory is unable to describe correctly the transition from a uniform movement of a charge into an arbitrary one and then again into uniform over a limited interval of time. In this case, the first and the latter solutions can be given exactly by the Lorentz transformation. Furthermore the question arises: what mechanism changes these potentials at the distance unreachable for retarded Liénard-Wiechert fields? The lack of continuity between the corresponding solutions is obvious. It has the same nature as discussed in the above sections, due to incompleteness of existent solutions.

The new approach also highlights the invariant deficiency of the self-energy concept in the framework of relativity theory. We confine our reasoning to the example of the electrostatic. The total potential energy of N charges due to all the forces acting between them is

$$W=\frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\frac{q_iq_j}{|r_i-r_j|}.\tag{32}$$

Here, the infinite self-energy terms $(i=j)$ are omitted in the double sum. The expression obtained by Maxwell for the energy in an electric field, expressed as a volume integral over the field, is [14]:

$$W=\frac{1}{2}\int_{\mathscr{V}}E^2d\mathscr{V}.\tag{33}$$

This corresponds to Maxwell's idea that the system energy must be stored somewhere in space. The expression (33) includes self-energy terms and in the case of point charges they make infinite contributions to the integral. The introduction of a finite radius for the elementary charges enables one to get rid of that difficulty but breaks down the possibility to see the classical electrodynamics as a self-consistent theory (Poincaré's non-electrical forces [15]).

In spite of introducing the self-energy concept long before the special relativity principle had arisen, there was not much alarm about the fact that it did not satisfy the relativity invariance condition. Strictly speaking, Einstein's theory refutes the invariance of energy. The law of energy conservation cannot be maintained in its classical form. In a relativistically covariant formulation the conservation of energy and the conservation of momentum are not independent principles. In particular, the local form of the energy-momentum conservation law can be written in a covariant form, using the energy-momentum tensor

$$\frac{\partial T^{\mu\nu}}{\partial x^{\nu}}=0.\tag{34}$$

For an electromagnetic field, it is well known that (34) can be strictly satisfied only for a free field (when a charge is not taken into account), whereas, for the total field of a charge this is not true, since (34) is not satisfied mathematically (four-dimensional analogy of Gauss's theorem). As everyone knows in classical electrodynamics, this fact gives rise to the "electromagnetic mass" concept, which violates the exact relativistic mass-energy relationship ($\mathscr{E}=mc^2$). Let us examine this problem in a less formal manner. The equivalent three-dimensional form of (34) is the formula (9). The amount of electrostatic energy of an unmoving charge in a given volume $\mathscr{V}$ is proportional to E^2 [see (33)]. According to (34) [or (9)], in a new inertial frame K', this value W must be, generally speaking, an explicit time-dependent function ($\partial w/\partial t\neq 0$). Furthermore, this means also the explicit time dependence for the electric field ($\partial\mathbf{E}/\partial t\neq 0$). On the other hand, the electric field strength of an unmoving charge keeps its implicit time-dependent behavior under the

Lorentz transformation ($\partial\mathbf{E}/\partial t=\mathbf{0}$). The conflict with the relativistic invariance condition is obvious. The analogous reasoning can be applied for Coulomb's electrostatic energy of a system of charged particles. In this case, if one is thinking that electrostatic energy can be stored locally in space, the conflict with the relativity principle is inevitable. However, in the framework of the above-stated separated-potential method it is possible to avoid those difficulties. Actually, in the new general solution (25) $\mathbf{E}_0$ is the only term exclusively linked to the charges. According to the above speculation, no local energy conservation law can be written for this field $\mathbf{E}_0$. The mathematical form (32) must be saved for it. But there is no cause to reject the local form for the time-dependent free field $\mathbf{E}^*$. In fact, the mathematical expression (33) is adequate for it. Thus, if one wishes not to get into trouble with the relativity principle, one must distinguish two different terms in the total electric field energy:

$$W=\frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\frac{q_iq_j}{|r_i-r_j|}+\frac{1}{2}\int_{\mathscr{V}}E^{*2}d\mathscr{V}. \tag{35}$$

We should make one further remark about this energy formula. In first place, the dualism concept reveals the dual nature of the electromagnetic field energy. So, for instance, the total electric energy is the electrostatic energy plus the electric energy of the free electromagnetic field. The first term is nonzero if the system consists of at least two interacting charged particles. The second term is taken as an integral over the region of $\mathscr{V}$ where the local value of E^* is not equal to zero. In the next section the correctness of this energy representation for all electromagnetic fields will be strictly verified by applying the principle of least action. The introduction of the self-energy concept in 19th century physics can be explained historically. Maxwell considered the total electromagnetic field to be a uniform physical object in its own right.

Removing the self-energy concept, a valuable mechanical analogy of the Maxwell equations in the form of (21)–(24) can be used to understand why their general solution must be as separated potentials (19), (20). From the mathematical point of view, the two equations (21) and (22) correspond to the electrostatic and magnetostatic approximations, respectively, and may be considered as wave equations with infinite spread velocity of longitudinal perturbations. If there is no local energy transfer, Einstein's theory does not limit the signal spreading velocities. In this case, the set of differential equations for elastic waves in an isotropic media (see [16]) can be treated as a mechanical analogy of (21)–(24):

$$\frac{\partial^2 u_{\ell}}{\partial t^2}-c_{\ell}^2\Delta u_{\ell}=0, \tag{36}$$

$$\frac{\partial^2 u_t}{\partial t^2}-c_t^2\Delta u_t=0. \tag{37}$$

The general solution of (36) and (37) is the sum of two independent terms corresponding to longitudinal u_{ℓ} and transverse u_t waves:

$$u=u_{\ell}+u_t. \tag{38}$$

If the longitudinal spreading velocity approaches formally to infinity ($c_{\ell}\rightarrow\infty$) then (36) transforms into Laplace's equation whereas the function u_{ℓ} turns out to have an implicit time dependence. Thus, the formula (38) takes the form of separated potential's solution (19) and (20).

To end this section, we note that the idea of nonlocal interactions can be immediately derived from Maxwell's equations as an exact mathematical result. On the other hand, some of the quantum mechanical effects like the Aharonov-Bohm effect [17], violation of the Bell's inequalities [18,19], etc., point out indirectly to the possibility of nonlocal interactions in electromagnetism. Nevertheless, in this work we prefer to confine themselves to the classical theory.

V. HAMILTONIAN FORM OF MAXWELL'S EQUATIONS FROM THE POINT OF VIEW OF SEPARATED POTENTIAL'S METHOD

In the last section we introduced the prototype for a new electromagnetic energy interpretation. In this section we shall discuss general field equations for arbitrary fields from the standpoint of the principle of least action and the change in their interpretation due to the new dualism concept. In extending the separated-potential method no modifications at all are necessary in the set of Maxwell's equations to make them agree with the requirements of the covariant formulation. Hence, in the steady approximation ($\varphi^*=0,\mathbf{A}^*=\mathbf{0}$) a relativistic action for a system of interacting charged particles can be written in the conventional form [9]

$$S_m+S_{mf}=\int\left(-\sum_{a=1}^{N}m_a c\, ds_a-\sum_{a=1}^{N}\frac{e_a}{c}\sum_{\mu=0}A_{0(\mu a)}dx_a^{\mu}\right), \tag{39}$$

where $A_{0(ma)}$ is the *instantaneous* potential ($\varphi_0,\mathbf{A}_0$) in the four point on the world line of the particle with the number "a" created by other particles. This expression is sufficient to derive the first couple of equations (21) and (22) from the least action principle. It can be proved directly rewriting the second term in (39) as

$$S_{mf}=-\frac{1}{c}\int\sum_{\mu}A_{0\mu}j^{\mu}d\mathscr{V}dt, \tag{40}$$

using the Dirac's expression for four-current:

$$j_{\mu}(\mathbf{r},t)=\sum_a\left[\frac{e_a}{4\pi}\Delta\left(\frac{1}{|\mathbf{r}-\mathbf{r}_a|}\right)\right]u_{\mu a}, \tag{41}$$

where $u_{\mu a}$ is the four-velocity of the charged particle "a." Generally, for a system of arbitrary moving charges, the time-dependent potentials ($\varphi^*,\mathbf{A}^*$) appear in the general solution. This means that an additional term corresponding to the free electromagnetic field must be added to (39). In the first place, it must vanish under the transition to the steady approximation ($\varphi^*=0,\mathbf{A}^*=0$). On the other hand, the variation of this term has to lead to the second pair of equations (23) and (24). As a result, it is easy to see that the conventional Hamiltonian form can be adopted to describe the presence of the free electromagnetic field [9]

$$S_f = -\frac{1}{16\pi}\int \sum_{\mu,\nu} F_{\mu\nu}F^{\mu\nu}d\mathscr{V}dt, \tag{42}$$

where

$$F_{\mu\nu} = \frac{\partial A^*_\nu}{\partial x^\mu} - \frac{\partial A^*_\mu}{\partial x^\nu}. \tag{43}$$

Finally, it remains to be proved that from the variation derivative,

$$\delta S_f = -\int \sum_\mu \left(\frac{1}{4\pi}\sum_\nu \frac{\partial F^{\mu\nu}}{\partial x^\nu}\right)\delta A^*_\mu d\mathscr{V}dt, \tag{44}$$

one obtains the covariant analog of (23) and (24) in the following form:

$$\sum_\nu \frac{\partial}{\partial x^\nu}F^{\mu\nu} = \sum_\nu \frac{\partial}{\partial x^\nu}\left[\frac{\partial A^{*\nu}}{\partial x_\mu} - \frac{\partial A^{*\mu}}{\partial x_\nu}\right] = 0. \tag{45}$$

The only difference with the classical field interpretation consists in the way electromagnetic potentials take part in this Hamiltonian formulation. Actually, the second term in (39) contains only instantaneous potentials whereas S_f is related with time-varying field components. Consequently, contrary to traditional interpretation, the quantity $F^{\mu\nu}$ can be defined as a free electromagnetic field tensor.

In light of the new approach, the electromagnetic energy-momentum tensor demands some corrections in the interpretation of its formal mathematical formulation [9]:

$$T^{\mu\nu} = -\frac{1}{4\pi}\sum_\rho F^\mu_\rho F^{\nu\rho} + \frac{1}{16\pi}g^{\mu\nu}\sum_{\beta,\gamma} F_{\beta\gamma}F^{\beta\gamma}. \tag{46}$$

As a consequence of (43), in this form it can describe the energy-momentum conservation law for, exclusively, the free electromagnetic field as follows:

$$\sum_\nu \frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0, \tag{47}$$

which supports the new interpretation of electric field energy given in the previous section. Strictly speaking, from the point of view of the dualism concept, the total field energy W must consist of two noncompatible parts: on one hand, the energy W_{mf} of electrostatic and magnetostatic interaction between charges and currents (*nonlocal* term), on the other hand, the energy W_f of the free electromagnetic field (*local* term):

$$W = W_{mf} + W_f. \tag{48}$$

This contradicts considerably the FMSI concept about the unique nature of electromagnetic field energy. Summarizing these results, we see that the concept of potential (*nonlocal*) energy and potential forces must be conserved as valid in classical electrodynamics. So, the system of charges and currents in the absence of free electromagnetic field must be considered as conservative system without any idealization. As an important remark we note the physical meaning of Poynting vector has been changed notably. So far the classical theory dealt with it as a quantity attributed to all dynamic properties of the total electromagnetic field. From the new point of view, it can be nonzero only in the presence of *free* field. The great problem of the classical electrodynamics, the indefiniteness in the location of the field energy, does not exist anymore. In particular, the flux of the electromagnetic energy in the steady state has no sense since no presence of the free electromagnetic field is supposed in this case.

VI. NONRADIATION CONDITION FOR FREE ELECTROMAGNETIC FIELD

In this section we shall discuss the energy balance between the system of interacting charged particles and free electromagnetic field, namely, energy and momentum lost by radiation. Turning to the results of the last section we must examine carefully one essential difference in electromagnetic energy interpretation. Let us write the total relativistic action as

$$S = S_m + S_{mf} + S_f. \tag{49}$$

Although we adopt the same notation used in the conventional theory, the physical essence of the last two terms has changed significantly. Usually, the interaction between particles and electromagnetic field was attributed to S_{mf} whereas the properties of electromagnetic field manifested themselves by the additional term S_f.

In the new approach, no concept of field as intermediary is needed to describe the interaction between charges (currents). Hence, S_{mf} cannot be treated in terms of particle-field interaction. Such interaction as well as the intrinsic properties of a free electromagnetic field are enclosed now in the last term S_f. The possible free field interaction with the system of charges (currents) depends entirely on its location in space. This reasoning makes it possible to consider the isolated system of charged particles and free field as consisting of two corresponding subsystems. Each of the subsystems may be completely independent if there is no mutual interaction (for instance, free electromagnetic field is located far from the given region of charges and currents). In the steady approximation the first subsystem (charges and currents) can be considered as conservative. In other words, it means the total Hamiltonian of the whole isolated system can be decomposed into two corresponding parts:

$$\mathscr{H} = \mathscr{H}_1 + \mathscr{H}_2, \tag{50}$$

where $\mathscr{H}_1$ is the Hamiltonian of the conservative system of charges and currents. It involves, apart from electrostatic and magnetostatic energy, mechanical energy of particles (corresponds to the action $S_m + S_{mf}$). $\mathscr{H}_2$ is the Hamiltonian of the free electromagnetic field (corresponds to the action S_f).

We recall here that in the relativistic case, the energy is the zero component of the momentum. However, if we deal with the isolated system, the total Hamiltonian is not time dependent and the energy conservation law as well as the momentum conservation may be treated independently. It is important to note that such separation into two subsystems is valid only in the new approach. The conventional interpretation of S_f did not allow us to consider it separately. Actually, in the steady approximation S_f was not zero, and corresponded to the self-energy of field [9]:

$$S_f = \int_{t_2}^{t_1} \mathscr{L}_f dt, \tag{51}$$

where

$$\mathscr{L}_f = \frac{1}{8\pi} \int_{\mathscr{V}} (E^2 - B^2) d\mathscr{V}. \tag{52}$$

Here **E** and **B** are the total electrostatic and magnetostatic field strengths, respectively. Thus, the fact that S_f is responsible solely for free field turns out to be a meaningful argument in separating into two subsystems. It is often possible to extract a large amount of information about the physical nature of the system using conservation laws, even when complete solutions cannot be obtained. Let us now consider the case when charges (currents) and free electromagnetic field are located in the same region and become interacting. Internal forces of mutual reaction between two subsystems are usually named as internal dissipative forces. They carry out the energy exchange inside the total isolated system. In terms of the Hamiltonian formalism it can be expressed as a corresponding Hamiltonian evolution (see, for instance, [20]):

$$\frac{d\mathscr{H}_{1,2}}{dt} = \frac{\partial \mathscr{H}_{1,2}}{\partial t} + \mathscr{P}_{1,2}^{\text{ext}} + \mathscr{P}_{1,2}^{\text{int}}, \tag{53}$$

where $\mathscr{P}_{1,2}^{\text{ext}}(\mathscr{P}_{1,2}^{\text{int}})$ is the power of the external (internal) forces acting on the two subsystems, respectively. In our case $\mathscr{P}_1^{\text{ext}}$ and $\mathscr{P}_2^{\text{ext}}$ appear as a result of the mutual interaction. On the other hand, any internal nonpotential force in the first subsystem can also cause energy dissipation ($\mathscr{P}_1^{\text{int}}$). Even in the absence of a real mechanical friction, other internal nonpotential forces (for example, inhomogeneous gyroscopic forces) can still act in this subsystem and dissipate energy. In other words, if initially there is no free electromagnetic field ($\mathscr{H}_2 = 0$), it can be created by internal nonpotential forces ($\mathscr{P}_1^{\text{int}}$) acting in the first subsystem ($\mathscr{H}_2$ is no longer zero). It means that energy is lost by radiation in the subsystem of charges and currents. In mathematical language the corresponding energy balance can be written as follows:

$$\frac{d}{dt}(\mathscr{H}_1 + \mathscr{H}_2) = \dot{\mathscr{H}}_1 + \dot{\mathscr{H}}_2 = 0, \tag{54}$$

where $\dot{\mathscr{H}}_1$ and $\dot{\mathscr{H}}_2$ are energy change rates for the first and the second subsystems, respectively. It might be easily noted that the energy balance (54) is symmetrical in respect to time reversion, which is in accordance with the time symmetry of Maxwell's equations. The real direction of the energy exchange process may be determined by some subsidiary conditions. On the contrary to this, the energy balance in the conventional electrodynamics was always irreversible in time. From the other hand, the former class of theories based on the action at a distance principle (for example, the electrodynamics of Wheeler and Feynman) did not consider at all the third term S_f in (49), corresponding to radiation reaction. As a matter of fact, there were no radiation effects in those theories, but only interactions of a number of particles.

To end the section we formulate the previous statement about the energy conservation as *the condition of nonradiation of the free electromagnetic field*.

If in an isolated system of charges (currents) in the absence of free electromagnetic field ($\mathscr{H}_2 = 0$), all internal nonpotential forces are compensated or do not exist then this system will not produce (radiate) free electromagnetic field ($\mathscr{H}_2$ remains zero) and will keep the conservative system itself.

This implies not only an equilibrium between radiation and absorption but no radiation at all. As a simple example of a nonradiating system we can consider two charged particles moving with acceleration along a direct line under mutual Coulomb interaction. The absence of other frictional forces is supposed. The presence of any inhomogeneous gyroscopic (Lorentz-type) forces here are not expected due to the one-dimensional character of motion. Some mention should also be made of the many-particle system. It is possible that there is some limited class of motion when all nonpotential (for example, internal gyroscopic forces) can be compensated due to the own magnetic moment of charged particles. This possibility would be of particular interest in the attempt to understand the quantum mechanics principles.

In the present work the question of interaction of free fields with sources (charges and currents) is given in a perfunctory manner and should be studied carefully. It should be compared with the older nonradiation theories based on the *extended* Dirac electron models (see, for instance [21],). Furthermore, emission, absorption and, for instance, scattering processes can be caused by the interaction of matter fields with the $B(3)$ spin field. It is created by transverse left- and right-circular polarized waves, as found by Evans and Vigier [22–25]. On the other hand, the existence of the longitudinal $B(3)$ field may hint on nonzero photon mass. Theoretical constructs of such a type were introduced and developed by Einstein, Schrödinger, Deser, de Broglie, and Vigier (see, e.g., [26]). However, relations between $B(3)$ and other longitudinal solutions of Maxwell equations, as well as the problem of photon mass, must be studied more carefully.

VII. CONCLUSIONS

Finally, we conclude that the FMSI concept could not give a complete and adequate description of the great variety of electromagnetic phenomena. It has been shown that another concept (the so-called *dualism concept*), consistent with the full set of Maxwell's equations, can be accepted as a correct description of electromagnetism. In other words, the new concept states that there is a *simultaneous* and *independent* coexistence of Newton instantaneous long-range (NILI) and Faraday-Maxwell short-range interactions (FMSI) which cannot be reduced to each other. The reasons are based on the mathematical method (so-called *separated-potential method*) proposed in this work for a complete general solution of Maxwell's equations. As a result, the incompleteness of former solutions of Maxwell's equations is proved.

In the framework of the new approach, all main concepts of the classical electrodynamics have been reconsidered. In particular, it has been shown that the dual nature of the total electromagnetic field must be taken into consideration. On one hand, there is a free electromagnetic field $\mathbf{E}^*(\mathbf{B}^*)$ that has no direct connection with charges and currents, and can be transferred *locally*. On the other hand, there is a field

$\mathbf{E}_0(\mathbf{B}_0)$ linked exclusively to charges (currents) and responsible for interparticle interaction, which *cannot be transferred locally* in space. However, in total, these two kinds of electromagnetic fields $\mathbf{E}_0+\mathbf{E}^*$ $(\mathbf{B}_0+\mathbf{B}^*)$ as a superposition satisfy Maxwell's equations and are observed experimentally as a unique electromagnetic field. Other quantities of the classical electrodynamics such as electromagnetic field tensor, electromagnetic energy-momentum tensor, etc., have also changed their physical meanings. In particular, the Poynting vector can be associated *only with the free electromagnetic field*. In light of this result, the problem of the indefiniteness in the field energy location has no place and no flux of electromagnetic energy in steady state can be derived from the theory. Also problems such as *self-force, infinite contribution of self-energy, the concept of electromagnetic mass, and radiation irreversibility in time with respect of Maxwell equations* have been removed in the new approach. A new interpretation of the energy conservation law is possible as a nonradiation condition that states that *a limited class of motion exists when accelerated charged particles do not produce electromagnetic radiation*.

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